Optimal Power Flow using Interior Point Method in Polar Form

EE 8725 – Project Presentation

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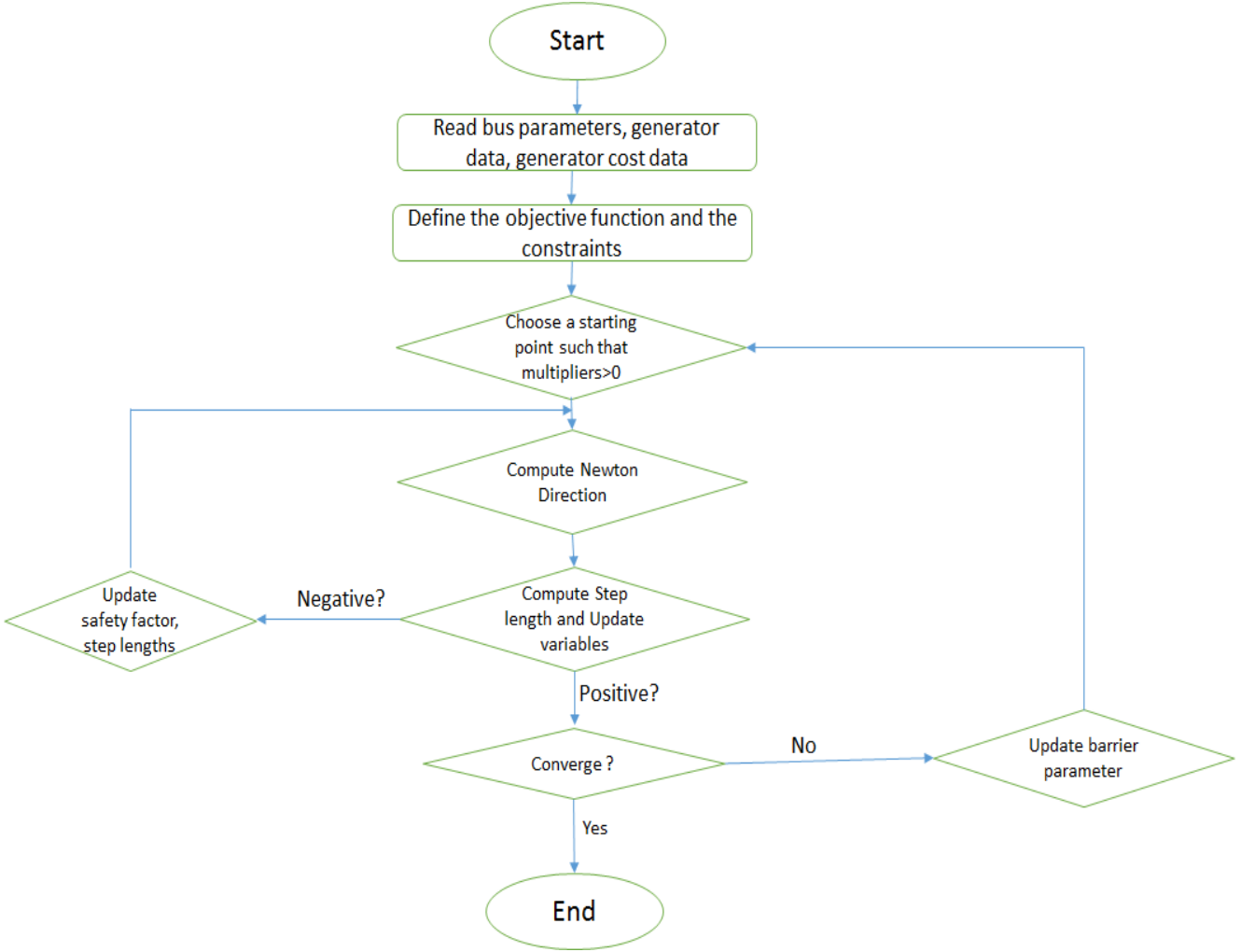
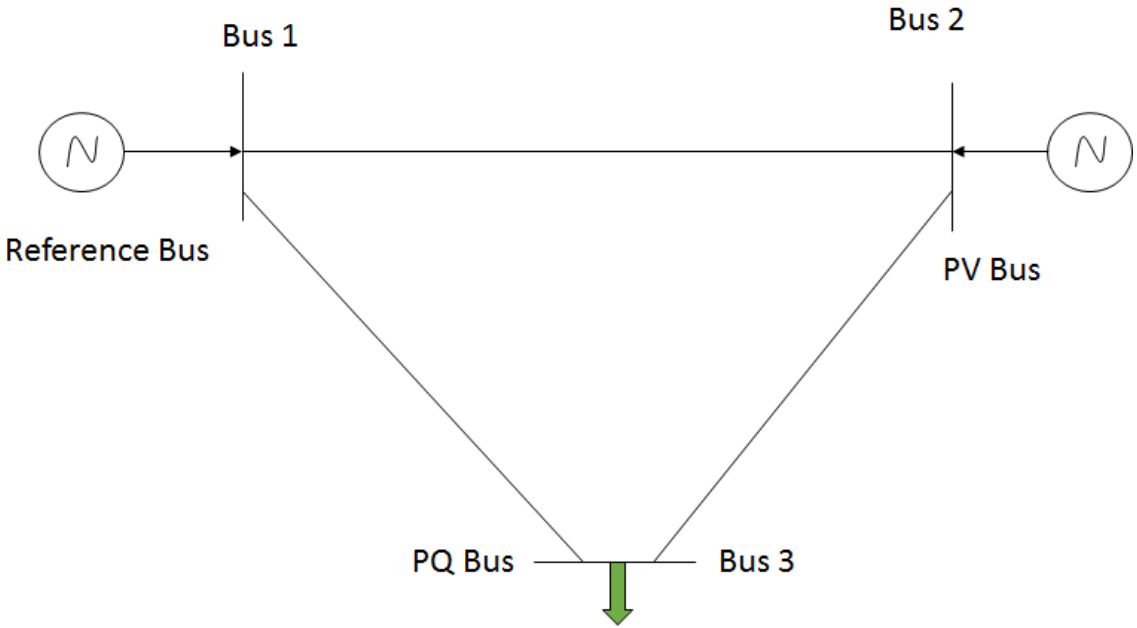
Outline

- Review OPF using IPM
- Expressions used to solve OPF in Polar coordinates
- Different matrices for a 3 bus test case
- Simulation results

Algorithm


Start
Read bus parameters, generator data, generator cost data
Define the objective function and the constraints
Choose a starting point such that multipliers>0
Compute Newton Direction
Compute Step length and Update variables
Negative?
Update safety factor, step lengths
Positive?
Converge ?
No
Update barrier parameter
Yes
End

3 bus test case



Data

```
%% bus data
%  bus type     Pd     Qd   Gs      Bs area Vm   Va   baseKV zone   Vmax      Vmin
bus = [
    1   3      0.0    0.0  0.0     0.0  1   1.0  0.0    345.0   1    1.1000   0.9000;
    2   2      0.0    0.0  0.0     0.0  1   1.0  0.0    345.0   1    1.1000   0.9000;
    3   1      100.0  20.0 0.0     0.0  1   1.0  0.0    345.0   1    1.1000   0.9000;
];
```

```
%% generator data
%  bus      Pg     Qg      Qmax      Qmin      Vsp  base status  Pmax       Pmin
gen = [
    1      20     0.0000   300.0   -300.0000   1.0   100.0   1   200.0000    20.0000;
    2    100.0    0.0000   300.0   -300.0000   1.0   100.0   1   100.0000    10.0000;
];
```

```
%% branch data
%fbus tbus    r        x        b      ratea    rateb     ratec      ratio angle  status
branch = [
    1    2   0.0200   0.0576   0.0200   250.0    250.0    250.0000     0.0   0.0     1;
    1    3   0.0170   0.0920   0.1580   250.0    250.0    250.0000     0.0   0.0     1;
    2    3   0.0390   0.1700   0.3580   150.0    150.0    150.0000     0.0   0.0     1;
];
```

Objective function f(x)

Minimize:

$$f(\boldsymbol{x}) = \sum_{i=1}^{n_g} C_i(P_{Gi}) = \sum_{i=1}^{n_g} \left[a_i + b_i \cdot P_{Gi} + c_i \cdot P_{Gi}^2\right]$$

Here,
a_i, b_i and c_i are cost coefficients of the real power of the generator,
n_g is the number of generators in the power system including the slack bus

Objective function and cost coefficients matrices

```
% C(P) = a + b*PMW + c*PMW^2 :
% PMW = baseMVA * Ppu
% C(P) = a + b*(baseMVA*Ppu) + c*(baseMVA*Ppu)^2 :
% C = [c b a]
%% additional changes will be made
C_temp  = gencost(:,5:7); % Coefficients of F(P) polynomials
% C_temp(1:size(C_temp,1),:)
L.C  = [(P.baseMVA^2)*C_temp(:,1) (P.baseMVA)*C_temp(:,2) C_temp(:,3)];
L.dC = [2*L.C(:,1) L.C(:,2)];     % Coefficients of dF(P) polynomials
```

	1	2	3
1	1100	500	150
2	850.0000	120	600

L.C
(2 X 3)

	1	2
1	2200	500
2	1.7000e+03	120

L.dC
(2 X 2)

State variables and equality constraints

$$\boldsymbol{x} = \left[\delta_2, \delta_3, \ldots, \delta_N, |V_2|, |V_3|, \ldots, |V_N|, P_{G1}, P_{G2}, \ldots, P_{Gn_g}, Q_{G1}, Q_{G2}, \ldots, Q_{Gn_g}\right]^T$$

1. Load constraints where P_{Lk} and O_{Lk} are fixed:

$$\sum_{m=1}^{N}\left[|V_k|\sum_{m=1}^{N}|Y_{km}||V_m|\cos(\psi_{km})\right] + P_{Lk} = \mathbf{0}$$

$$\sum_{m=1}^{N}\left[|V_k|\sum_{m=1}^{N}|Y_{km}||V_m|\sin(\psi_{km})\right] + Q_{Lk} = \mathbf{0}$$

2. Generation constraints where P_{Gk} and Q_{Gk} are not fixed:

$$\sum_{m=1}^{N}\left[|V_k|\sum_{m=1}^{N}|Y_{km}||V_m|\cos(\psi_{km})\right] - P_{Gk} = \mathbf{0}$$

$$\sum_{m=1}^{N}\left[|V_k|\sum_{m=1}^{N}|Y_{km}||V_m|\sin(\psi_{km})\right] - Q_{Gk} = \mathbf{0}$$

3. Voltage magnitude equality constraints:

$$|V_{Gk}| = Constant$$

Inequality constraints

$$\boldsymbol{h}(\boldsymbol{x}) = \begin{bmatrix} P_{Gk}^{Min} \leq P_{Gk} \leq P_{Gk}^{Max} \\ Q_{Gk}^{Min} \leq Q_{Gk} \leq Q_{Gk}^{Max} \\ |V_k|^{Min} \leq |V_k| \leq |V_k|^{Max} \\ P_{km}^{Min} \leq P_{km} \leq P_{km}^{Max} \end{bmatrix}$$

$$\boldsymbol{h}(\boldsymbol{x}) = \begin{bmatrix} [P_{Gk}]^{Max} \\ [P_{Gk}]^{Min} \\ [Q_{Gk}]^{Max} \\ [Q_{Gk}]^{Min} \\ [|V_k|]^{Max} \\ [|V_k|]^{Min} \\ [P_{km}]^{Max} \\ [P_{km}]^{Min} \end{bmatrix} = \begin{bmatrix} P_{Gk} - P_{Gk}^{Max} \\ P_{Gk}^{Min} - P_{Gk} \\ Q_{Gk} - Q_{Gk}^{Max} \\ Q_{Gk}^{Min} - Q_{Gk} \\ |V_k| - |V_k|^{Max} \\ |V_k|^{Min} - |V_k| \\ P_{km} - P_{km}^{Max} \\ P_{km}^{Min} - P_{km} \end{bmatrix} \leq 0$$

Gradients

$$\nabla_x f(\boldsymbol{x}) = \begin{bmatrix} \frac{\partial f}{\partial x_1} & \frac{\partial f}{\partial x_2} & \cdots & \frac{\partial f}{\partial x_{n_v}} \end{bmatrix}^T$$

$$\nabla_x \boldsymbol{g}(\boldsymbol{x}) = \begin{bmatrix} \frac{\partial g_1}{\partial x_1} & \frac{\partial g_2}{\partial x_1} & \cdots & \frac{\partial g_{n_{ceq}}}{\partial x_1} \\ \frac{\partial g_1}{\partial x_2} & \frac{\partial g_2}{\partial x_2} & \cdots & \frac{\partial g_{n_{ceq}}}{\partial x_2} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial g_1}{\partial x_{n_v}} & \frac{\partial g_2}{\partial x_{n_v}} & \cdots & \frac{\partial g_{n_{ceq}}}{\partial x_{n_v}} \end{bmatrix}$$

$$\nabla_x \boldsymbol{h}(\boldsymbol{x}) = \begin{bmatrix} \frac{\partial h_1}{\partial x_1} & \frac{\partial h_2}{\partial x_1} & \cdots & \frac{\partial h_{n_c}}{\partial x_1} \\ \frac{\partial h_1}{\partial x_2} & \frac{\partial h_2}{\partial x_2} & \cdots & \frac{\partial h_{n_c}}{\partial x_2} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial h_1}{\partial x_{n_v}} & \frac{\partial h_2}{\partial x_{n_v}} & \cdots & \frac{\partial h_{n_c}}{\partial x_{n_v}} \end{bmatrix}$$

Grad:

$$\nabla_x \boldsymbol{h}(\boldsymbol{x}) = \begin{bmatrix} [\nabla_x P_G]^{Max} & [\nabla_x P_G]^{Min} & [\nabla_x Q_G]^{Max} & [\nabla_x Q_G]^{Min} \\ [\nabla_x |V_k|]^{Max} & [\nabla_x |V_k|]^{Min} & [\nabla_x P_{km}]^{Max} & [\nabla_x P_{km}]^{Min} \end{bmatrix}$$

$$[\nabla_x P_G]^{Max} = \begin{bmatrix} \frac{\partial [P_{G_k}]^{Max}}{\partial V_k} \\ \frac{\partial [P_{G_k}]^{Max}}{\partial P_{G_k}} \\ \frac{\partial [P_{G_k}]^{Max}}{\partial Q_{G_k}} \end{bmatrix} = \begin{bmatrix} \mathbf{0} & (u \times n_g) \\ \mathbf{1} & (n_g \times n_g) \\ \mathbf{0} & (n_g \times n_g) \end{bmatrix}$$

$$[\nabla_x Q_G]^{Max} = \begin{bmatrix} \frac{\partial [Q_{G_k}]^{Max}}{\partial V_k} \\ \frac{\partial [Q_{G_k}]^{Max}}{\partial P_{G_k}} \\ \frac{\partial [Q_{G_k}]^{Max}}{\partial Q_{G_k}} \end{bmatrix} = \begin{bmatrix} \mathbf{0} & (u \times n_g) \\ \mathbf{0} & (n_g \times n_g) \\ \mathbf{1} & (n_g \times n_g) \end{bmatrix}$$

$$[\nabla_x |V_k|]^{Max} = \begin{bmatrix} \frac{\partial [|V_k|]^{Max}}{\partial V_k} \\ \frac{\partial [|V_k|]^{Max}}{\partial P_{G_k}} \\ \frac{\partial [|V_k|]^{Max}}{\partial Q_{G_k}} \end{bmatrix} = \begin{bmatrix} Nonzero & (u \times n_g) \\ \mathbf{0} & (n_g \times n_g) \\ \mathbf{0} & (n_g \times n_g) \end{bmatrix}$$

$$[\nabla_x P_{km}]^{Max} = \begin{bmatrix} \frac{\partial [P_{km}]^{Max}}{\partial V_k} \\ \frac{\partial [P_{km}]^{Max}}{\partial P_{G_k}} \\ \frac{\partial [P_{km}]^{Max}}{\partial Q_{G_k}} \end{bmatrix} = \begin{bmatrix} Nonzero & (u \times n_g) \\ \mathbf{0} & (n_g \times n_g) \\ \mathbf{0} & (n_g \times n_g) \end{bmatrix}$$

$$[\nabla_x P_G]^{Min} = -[\nabla_x P_G]^{Max}$$

$$[\nabla_x Q_G]^{Min} = -[\nabla_x Q_G]^{Max}$$

$$[\nabla_x |V_k|]^{Min} = -[\nabla_x |V_k|]^{Max}$$

$$[\nabla_x P_{km}]^{Min} = -[\nabla_x P_{km}]^{Max}$$

Gradient matrices

Grad of obj func	
(5,1)	939.9992
(6,1)	290

Gradient of f(x)
(8 X 1)

Gradient of equality constraints
(8 X 6)
28 full

Grad of Eq	
(1,1)	-5.1297
(2,1)	14.1508
(3,1)	-2.5766
(4,1)	9.9119
(5,1)	-1
(1,2)	6.1615
(2,2)	19.0723
(3,2)	-0.7716
(4,2)	-4.9216
(6,2)	-1
(1,3)	-1.6311
(2,3)	-4.7508
(3,3)	2.0342
(4,3)	14.6626
(1,4)	-15.5777
(2,4)	-4.6598
(3,4)	-10.3734
(4,4)	-2.462
(7,4)	-1
(1,5)	16.9617
(2,5)	-5.3971
(3,5)	-5.1508
(4,5)	0.7372
(8,5)	-1
(1,6)	-5.2298
(2,6)	1.4817
(3,6)	14.9268
(4,6)	-3.9437

Grad of Ineq	
(5,1)	1
(6,2)	1
(5,3)	-1
(6,4)	-1
(7,5)	1
(8,6)	1
(7,7)	-1
(8,8)	-1
(1,9)	1.8168
(3,10)	1.911
(1,11)	-1.8168
(3,12)	-1.911
(1,13)	-5.6281
(2,13)	-13.9936
(3,14)	-1.3006
(4,14)	-10.1371
(1,15)	1.5176
(2,15)	4.9216
(3,15)	-0.7716
(4,15)	-4.9216
(1,16)	5.6281
(2,16)	13.9936
(3,17)	1.3006
(4,17)	10.1371
(1,18)	-1.5176
(2,18)	-4.9216
(3,18)	0.7716
(4,18)	4.9216

- Gradient matrices are sparse

Gradient of inequality constraints
(8 X 18)
28 full

Optimality conditions

$$\nabla_x L_\mu = \nabla_x f(x) + \nabla_x g(x) \,.\, \lambda + \nabla_x h(x) \,.\, \gamma = 0$$

$$\nabla_\lambda L_\mu = g(x) = 0$$

$$\nabla_\gamma L_\mu = h(x) + s = 0$$

$$\nabla_s L_\mu = -\mu^k \,.\, S^{-1} \,.\, e + \gamma = 0$$

```
grad_L_x      = grad_f + grad_g*lambda + grad_h*gamma;
grad_L_lambda = ceq;
grad_L_gamma  = c + s;
S_inv = spdiags(1./s,0,LNC,LNC); % Inverse of S Matrix
grad_L_s      = -mu*(S_inv)*e + gamma;
```

Gradient of Lagrangian

	A	B	C	D
1	**grad_L_gamma**	**grad_L_lambda**	**grad_L_s**	**grad_L_x**
2	**18*1**	**6*1**	**18*1**	**8*1**
3	-7.54E-09	-2.26E-09	5.19E-05	-3.08E-08
4	9.43E-09	-1.06E-08	0.000103806	2.32E-07
5	9.43E-09	1.12E-08	-42.82896441	5.54E-08
6	9.43E-10	-1.87E-11	154.6916484	1.40E-07
7	-1.89E-08	1.87E-09	8.14E-05	8.85E-06
8	-1.89E-08	-1.20E-10	1.93E-05	1.71E-05
9	-1.89E-08		1.93E-05	-9.43E-09
10	-1.89E-08		8.00E-05	-9.43E-09
11	7.58E-09		0.000242765	
12	7.48E-09		0.000314523	
13	7.50E-09		0.006144049	
14	7.60E-09		0.000906807	
15	-1.31E-08		4.18E-05	
16	-1.45E-08		5.19E-05	
17	-5.70E-09		7.92E-05	
18	-1.52E-08		3.38E-05	
19	-1.37E-08		2.92E-05	
20	-3.72E-09		5.13E-05	

Hessian

$\nabla_x^2 L_\mu = \nabla_x^2$ f(x) + $\sum_{i=1}^{nceq} \boldsymbol{\lambda}_i \,.\, \nabla_x^2\, g_i(x) + \sum_{j=1}^{nc} \gamma_j \,.\, \nabla_x^2\, h_j(x)$

$$\nabla_x^2 f(\boldsymbol{x}) = \boldsymbol{H}_f = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} & \cdots & \frac{\partial^2 f}{\partial x_1 \partial x_{n_v}} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} & \cdots & \frac{\partial^2 f}{\partial x_2 \partial x_{n_v}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 f}{\partial x_{n_v} \partial x_1} & \frac{\partial^2 f}{\partial x_{n_v} \partial x_2} & \cdots & \frac{\partial^2 f}{\partial x_{n_v}^2} \end{bmatrix}$$

$$\nabla_x^2 g_i(\boldsymbol{x}) = \boldsymbol{H}_{g_i} = \begin{bmatrix} \frac{\partial^2 g_i}{\partial x_1^2} & \frac{\partial^2 g_i}{\partial x_1 \partial x_2} & \cdots & \frac{\partial^2 g_i}{\partial x_1 \partial x_{n_v}} \\ \frac{\partial^2 g_i}{\partial x_2 \partial x_1} & \frac{\partial^2 g_i}{\partial x_2^2} & \cdots & \frac{\partial^2 g_i}{\partial x_2 \partial x_{n_v}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 g_i}{\partial x_{n_v} \partial x_1} & \frac{\partial^2 g_i}{\partial x_{n_v} \partial x_2} & \cdots & \frac{\partial^2 g_i}{\partial x_{n_v}^2} \end{bmatrix}$$

$$\nabla_x^2 h_i(\boldsymbol{x}) = \boldsymbol{H}_{h_i} = \begin{bmatrix} \frac{\partial^2 h_i}{\partial x_1^2} & \frac{\partial^2 h_i}{\partial x_1 \partial x_2} & \cdots & \frac{\partial^2 h_i}{\partial x_1 \partial x_{n_v}} \\ \frac{\partial^2 h_i}{\partial x_2 \partial x_1} & \frac{\partial^2 h_i}{\partial x_2^2} & \cdots & \frac{\partial^2 h_i}{\partial x_2 \partial x_{n_v}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 h_i}{\partial x_{n_v} \partial x_1} & \frac{\partial^2 h_i}{\partial x_{n_v} \partial x_2} & \cdots & \frac{\partial^2 h_i}{\partial x_{n_v}^2} \end{bmatrix}$$

Hessian matrices

Hess of obj func	
(5,5)	2200
(6,6)	1700

Hessian of f(x)
(8 X 8)

Hessian of equality constraints
(8 X 48)
36 full

Hcg	
(3,9)	1.1298
(4,9)	10.1558
(2,10)	-1.6027
(3,10)	-9.6551
(4,10)	1.0263
(1,11)	1.1298
(2,11)	-9.6551
(1,12)	10.1558
(2,12)	1.0263
(3,17)	1.7071
(4,17)	-5.2298
(3,18)	4.972
(4,18)	1.6311
(1,19)	1.7071
(2,19)	4.972
(1,20)	-5.2298
(2,20)	1.6311
(4,20)	-1.1705
(3,33)	10.6287
(4,33)	-1.0796
(2,34)	-8.6733
(3,34)	1.0263
(4,34)	9.2255
(1,35)	10.6287
(2,35)	1.0263
(1,36)	-1.0796
(2,36)	9.2255
(3,41)	5.4734
(4,41)	1.6311
(3,42)	-1.5507
(4,42)	4.7508
(1,43)	5.4734
(2,43)	-1.5507
(1,44)	1.6311
(2,44)	4.7508
(4,44)	-5.102

Hessian of inequality constraints
(8 X 144)
28 full

Hch	
(1,65)	2
(2,66)	2
(3,75)	2
(4,76)	2
(1,81)	-2
(2,82)	-2
(3,91)	-2
(4,92)	-2
(1,113)	2.564
(3,113)	-0.8494
(4,113)	-5.4179
(2,114)	0.7372
(3,114)	5.1508
(4,114)	-0.7372
(1,115)	-0.8494
(2,115)	5.1508
(1,116)	-5.4179
(2,116)	-0.7372
(1,137)	-2.564
(3,137)	0.8494
(4,137)	5.4179
(2,138)	-0.7372
(3,138)	-5.1508
(4,138)	0.7372
(1,139)	0.8494
(2,139)	-5.1508
(1,140)	5.4179
(2,140)	0.7372

Solving for Newton direction

$$\underbrace{\begin{bmatrix} \nabla_x^2 L_\mu & \nabla_x g(x) & \nabla_x h(x) & 0 \\ \nabla_x g(x)^T & 0 & 0 & 0 \\ \nabla_x h(x)^T & 0 & 0 & I \\ 0 & 0 & I & \nabla_s^2 L_\mu \end{bmatrix}}_{\mathrm{A}} \cdot \underbrace{\begin{bmatrix} \Delta x \\ \Delta \lambda \\ \Delta \gamma \\ \Delta s \end{bmatrix}}_{\mathrm{x}} = - \underbrace{\begin{bmatrix} \nabla_x L_\mu \\ \nabla_\lambda L_\mu \\ \nabla_\gamma L_\mu \\ \nabla_s L_\mu \end{bmatrix}}_{\mathrm{b}}$$

```
Delta = M\b;
delta_x      = Delta((1:LNV),1);
delta_lambda = Delta((LNV+1:LNV+LNCEQ),1);
delta_gamma  = Delta((LNV+LNCEQ+1:LNV+LNCEQ+LNC),1);
delta_s      = Delta((LNV+LNCEQ+LNC+1:LNV+LNCEQ+2*LNC),1);
```

Newton Direction

	A	B	C	D
1	**delta_x**	**delta_lambda**	**delta_gamma**	**delta_s**
2	**8*1**	**6*1**	**18*1**	**18*1**
3	-1.21E-08	0.001699036	-7.37E-05	1.22E-07
4	-3.50E-09	-0.001701223	-0.000147275	1.08E-07
5	-5.80E-09	-0.001695649	-0.001846722	-1.37E-07
6	-7.51E-09	-8.83E-05	0.001378353	-1.88E-07
7	-6.45E-08	8.60E-05	-0.000115519	-1.38E-07
8	-1.81E-07	0.000292527	-2.74E-05	2.91E-07
9	2.83E-07		-2.73E-05	4.27E-07
10	-1.47E-07		-0.000113537	-2.53E-09
11			-0.000344316	-3.62E-08
12			-0.000446019	-4.62E-08
13			-0.008668548	-7.94E-08
14			-0.001284615	-6.93E-08
15			-5.94E-05	-1.67E-08
16			-7.37E-05	2.77E-08
17			-0.000112434	3.78E-08
18			-4.79E-05	2.33E-07
19			-4.15E-05	1.89E-07
20			-7.29E-05	3.44E-08

Updating variables

$$x^{k+1} = x^k + k_s \,.\, \alpha_P \,.\, \Delta x$$

$$s^{k+1} = s^k + k_s \,.\, \alpha_P \,.\, \Delta s$$

$$\lambda^{k+1} = \lambda^k + k_s \,.\, \alpha_D \,.\, \Delta \lambda$$

$$\gamma^{k+1} = \gamma^k + k_s \,.\, \alpha_D \,.\, \Delta \gamma$$

```
L.x      = L.x      + ks*alphaDP*delta_x;
lambda   = lambda   + ks*alphaDP*delta_lambda;
gamma    = gamma    + ks*alphaDP*delta_gamma;
s        = s        + ks*alphaDP*delta_s;
```

•Scalars α_P and α_D are the step length parameters.

•Scalar $k_s \in (0, 1)$ represent the safety factor that guarantees the strict positive condition of the slack variables s and γ during each iteration.

Primal and dual step length

$$\alpha_{PMax} = \min \left[\min_{\Delta s_i < 0} \frac{s_i}{|\Delta s_i|}, 1 \right]$$

$$\alpha_{DMax} = \min \left[\min_{\Delta \gamma_i < 0} \frac{\gamma_i}{|\Delta \gamma_i|}, 1 \right]$$

```
SEARCH_PRIMAL = find(delta_s < 0);
SEARCH_DUAL = find(delta_gamma < 0);
% Primal step length parameter
alphaPMAX = min( min(s(SEARCH_PRIMAL)./abs(delta_s(SEARCH_PRIMAL))), 1 );
% Dual step length parameter
alphaDMAX = min( min(gamma(SEARCH_DUAL)./abs(delta_gamma(SEARCH_DUAL))), 1 );
```

Updating μ

- $\rho^k = (\gamma^k)^T \,.\, s^k$
- $\mu^k = \sigma \,.\, \frac{\rho^k}{n_c}$

Here γ^k and s^k are column vectors calculated at each iteration k.

```
sigma_new = max((0.99)*sigma_null,0.10);
rho = (gamma)' * s;              % The complementarity gap
mu  = sigma_new * (rho/L.Nc);      % The barrier parameter
sigma_null = sigma_new;
```

Convergence check

$$v_1^k = \max[\max(h(x^k), \|g(x^k)\|$$

$$v_2^k = \frac{\|\nabla_x f(x^k) + \nabla_x g(x^k).\lambda^k + \nabla_x h(x^k).\gamma^k\|}{1 + \|x^k\|}$$

$$v_3^k = \frac{\rho^k}{1 + \|x^k\|}$$

$$v_4^k = \mu^k$$

```
v1 = max( max(c) , norm(ceq,inf) );
v2 = norm(grad_L_x,inf)/(1+norm(L.x));
v3 = rho/(1+norm(L.x));
v4 = mu;
```

Test Results

Iteration	F(x)	v1	v2	v3	v4
#		Error1	Error2	Error3	Error4
1	1864	1	1	1	1
2	868.82	0.41352	247.06	3.0064	0.057114
3	803.09	0.32973	190.4	2.4957	0.04849
4	795.37	0.31788	182.56	2.4089	0.046587
5	792.71	0.31015	176.8	2.3418	0.045169
6	783.99	0.25661	137.45	1.8734	0.03799
7	793.29	0.16905	81.033	1.1824	0.026284
8	797.2	0.15646	73.956	1.0924	0.024375
9	801.48	0.1487	69.913	1.0397	0.02309
10	843.36	0.087216	39.361	0.63581	0.014485
11	904.43	0.012082	4.9209	0.16567	0.0038997
12	911.79	0.0032273	1.3045	0.086694	0.0020297
13	912.87	0.0019449	0.78549	0.057858	0.001342
14	914.29	0.00025411	0.1024	0.015203	0.00034941
15	914.45	6.60E-05	0.026587	0.0083167	0.00018925
16	914.45	5.81E-05	0.023403	0.0071735	0.0001616
17	914.49	7.58E-06	0.0030535	0.00168	3.75E-05
18	914.5	9.89E-07	0.00039841	0.00083848	1.85E-05
19	914.5	8.25E-07	0.0003322	0.00069147	1.51E-05

Converged in 19 iterations in 0.34 Seconds				
How many?		How much?	P (MW)	Q (MVAR)
Buses	3	Total Gen capacity	300	600
Generators	2	Generation (Current)	30	2.01
Loads	1	Load	100	20
Branches	3	Branch charging (inj)	-	48.15
Transformers	0	Shunt (inj)	0	0
Areas	1			

Generator Cost Data								
Gen.	Bus	C(P)=a + b*P + c*P^2			Pmin (MW)	Pmax (MW)	Gen.Cost	PG
#	#	a	b	c	$/h	(MW)		
1	1	150	5	0.11	20	200	294	20
2	2	600	1.2	0.085	10	100	620.5	10
						Total	914.5	30

Bus Data										
Bus	Voltage		Generation		Generation Limit		Load		Lambda($/MW-hr)	
#	Mag(pu)	Ang(deg)	P (MW)	Q (MVAR)	P (MW)	Q (MVAR)	P (MW)	Q (MVAR)	P	Q
1	1	0	20	185.23	200	300	-	-	0	-
2	0.909	1.014	10	-183.22	100	300	-	-	0	-
3	0.957	-3.637	-	-	-	-	100	20	0	-
		Total:	30	2.01	300	600	100	20		

Line Flow													
Bus	Voltage		Generation		Load		To Bus	Flow		Price ($/hr)			
#	Mag(pu)	Ang(deg)	P (MW)	Q (MVAR)	P (MW)	Q (MVAR)	#	P (MW)	Q (MVAR)	P	Q	deltaP	deltaQ
1	1	0	20	185.23	-	-				294			
							2	24.36	149.573				
							3	72.479	27.07			-76.839	8.583
2	0.909	1.014	10	-183.22	-	-				620.5			
							1	-19.707	-137.998				
							3	34.092	-47.042			-4.385	1.823
3	0.957	-3.637	-	-	100	20							
							1	-71.378	-36.254				
							2	-33.051	20.396			4.429	-4.142

Branch Data								
From Bus	To Bus	Line Limit P (MW)	From Bus Injection P (MW)	From Bus Injection Q (MVAR)	To Bus Injection P (MW)	To Bus Injection Q (MVAR)	Pkm + Pmk P (MW)	Qkm + Qmk Q (MVAR)
1	2	250	24.36	149.57	-19.71	-138	4.653	11.576
1	3	250	72.48	27.07	-71.38	-36.25	1.101	-9.184
2	3	150	34.09	-47.04	-33.05	20.4	1.041	-26.646
						Total:	6.795	-24.254

Disadvantages of IPM for OPF

The pure primal-dual interior-point algorithm was historically the first one used to solve OPF problems. It suffers, nevertheless, from three drawbacks:

- the heuristic to decrease the barrier parameter
- the required positivity of slack variables and their corresponding dual variables at every iteration, which may drastically shorten the Newton step length(s).
- more elaborate starting point choices can prevent negative variables from becoming too close to zero at an early stage, a condition that degrades IPM performance

In order to mitigate or to remove one or both among these flaws, two classes of methods emerged: higher-order IPMs (e.g., the predictor–corrector) and non-interior point methods (e.g., the Jacobian smoothing method).

Summary

- In daily operation of power systems, deciding an optimal control action, aiming at economic and reliable operation of a system, is an extremely difficult task. OPF problem is inevitably a very large non-convex NLP problem
- There is an increasing need to speed up solutions and IP methods have computationally proven to be viable alternatives for their solution.
- Tests performed under MATLAB confirm that IPM can directly and efficiently solve nonlinear OPF problems with good computational performance.

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THANK YOU !